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Stress Concentration at Inclined Crossbore Holes in Pressurized, Autofrettaged Tubes

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ARMAMENT RESEARCH, DEVELOPMENT AND ENGINEERING CENTER
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ABSTRACT

A sequence of analytic, elastic superpositions is employed to obtain a single equation for determining the maximum stress at the bore of a pressurized thick cylinder intersected by an inclined, circular crossbore hole whose axis intersects that of the cylinder. The crossbore hole may contain a pressure that differs from that in the bore, a situation that pertains in some gun tubes during firing.

The formulation permits the modeling of any plane (constant strain) end condition, including zero strain, open-ends and closed-ends. The crossbore hole may itself have an additional stress concentrator on its boundary; such concentrators may result from erosion, corrosion or manufacturing defects. Finally, preexisting residual stresses, prior to introduction of the crossbore hole, may also be incorporated

The single equation presented applies to the crossbore-bore intersection, but the formulation may be directly extended to model any radial location within the wall.

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INTRODUCTION

Crossbore holes are frequently introduced into thick cylinders, including gun tubes. When the center-line of a small circular crossbore hole coincides with the center-line of the tube, the intersection at the bore of the tube creates an ellipse, Figure 1. The eccentricity of the ellipse depends upon the angle of inclination of the hole centre-line to the axis of the tube, γ. The ratio major axis/minor axis is then given by $1/(\sin\gamma)$. This creates a significant stress concentration at a the end of the major axis, a location that is prone to thermal cracking and subsequent cyclic fatigue crack growth.

Cheng [1] reviewed available work and incorporated results due to Little & Bagci [2] which provide the stress concentration factor (SCF) at the bore intersection for the case in which full bore pressure also acts upon the crossbore hole. However, there is evidence that, because of a shock wave at the intersection, only a proportion of bore pressure infiltrates the crossbore in a gun tube [3], [4].

In this paper the Little and Bagci formulation is extended to include any ratio of pressure in crossbore to pressure in bore (α), any plane (constant axial strain) end-condition and any combination of residual hoop and axial stress arising from prior autofrettage.

Evacuators are also prone to gas erosion and/or corrosion pitting. These serve to further increase the SCF. The formulation is further extended to incorporate such effects.

The procedure involves superposition of a series of analytic solutions. In all cases it is necessary to model the effect of pressure infiltrating the stress concentrators. It is also necessary to properly account for contributions arising from that part of any biaxial stress field parallel to the major axis of the stress concentrator.

ANALYSIS: SIMPLE ELLIPSE

The hoop stresses at the bore of a plain tube, subjected to internal pressure p with internal radius a and external radius b, are given by Lamé's equations [5]:

$$\sigma_\theta = p\frac{(b^2 + a^2)}{(b^2 - a^2)} \qquad (1)$$

as noted above, the eccentricity of the elliptical hole at the bore (designated c) is given by:

$$c = 1/(\sin\gamma) \qquad (2)$$

where γ is the inclination of the hole centre-line to the axis of the tube.

The maximum stress around an elliptical hole subjected to remote uniaxial stress σ_n normal to the major axis occurs at the end of the major axis and is given by [5]:

$$\sigma_{ellipse(n)} = (1+2c)\sigma_n \qquad (3)$$

This situation is illustrated, together with other relevant solutions, in Figure 2.

So for the case where only Lamé hoop stress (defined in eqn. (1)) is acting, the stress at the end of the major axis is:

$$\sigma_\theta = p(1+2c)\frac{(b^2 + a^2)}{(b^2 - a^2)} \qquad (4)$$

The stress at the end of the major axis of an elliptical hole subjected to remote uniaxial stress σ_t parallel to the major axis is given by [5]:

$$\sigma_{ellipse(t)} = -\sigma_t \qquad (5)$$

Depending upon end conditions, the internal pressure may create an axial stress in the tube, this is given by:

$$\sigma_z = \frac{\beta p a^2}{(b^2 - a^2)} \qquad (6)$$

where $\beta = 0$ (constant axial strain, open ends), $\beta = 1$ (constant axial strain, closed ends) and $\beta = 2\nu$ (zero axial strain).

From eqn. (5), for the case where only Lamé axial stress (defined in eqn. (6)) is acting, the hoop stress at the end of the major axis created by σ_z is :

$$\sigma_\theta = \frac{-\beta p a^2}{(b^2 - a^2)} \qquad (7)$$

The total hoop stress relating to the Lamé hoop and axial stresses is obtained by summing eqns (4) and (7). This is normalized using bore hoop stress in the original tube, eqn (1), to obtain the associated stress concentration factor, K_{Lame}:

$$K_{Lame} = p\frac{[(1+2c)(b^2 + a^2) - \beta a^2]}{(b^2 - a^2)} \qquad (8)$$

Now assume that pressure αp acts within the evacuator, where $0 \le \alpha \le 1$

In this case the hoop stress at the end of the major axis is given by Godfrey [6]:

$$\sigma_\theta = \alpha p(2c - 1) \qquad (9)$$

Note: the necessary superpositions are again shown in Figure2.

Hence the total stress arising from Lamé hoop stress plus Lamé axial stress plus pressure infiltrating the evacuator may be obtained by summing eqns (4), (7) and (9). If this total stress is normalized using Lamé bore hoop stress from eqn (1) the stress concentration factor, $K_{Lame+pressure}$, is given by:

$$K_{Lame+pressure} = \frac{b^2[(2c+1) + \alpha(2c-1)] + a^2[(2c+1-\beta) + \alpha(2c-1)]}{(b^2 + a^2)} \qquad (10)$$

Equation (10) reduces to Cheng's eqn (10) [1], for $\alpha = 1$ (full pressure in evacuator), $\beta = 1$ (closed ends) and to Cheng's eqn (11) for $\alpha = 1$ (full pressure in evacuator), $\beta = 0$ (open ends). Obviously, it further reduces to:

$$K_{Lame+pressure} = (2c+1) \qquad (11)$$

for $\alpha = 0$ (no pressure in evacuator), $\beta = 0$

So, for the following open-end cases with $b/a = 2, c = 2$

Open end, full bore pressure in evacuator $(\alpha = 1, \beta = 0)$, $K_{Lame+pressure} = 6.8$

Open end, zero pressure in evacuator $(\alpha = 0, \beta = 0)$, $K_{Lame+pressure} = 5.0$

Open end, 20% of bore pressure in evacuator $(\alpha = 0.2, \beta = 0)$, $K_{Lame+pressure} = 5.36$

If there were pre-existing hoop and axial residual stresses $\sigma_{\theta R}$ and σ_{zR} within the tube, these may be treated in the same fashion as the Lamé stresses. This gives

$$\sigma_{\theta Rconc} = (1+2c)\sigma_{\theta R} - \sigma_{zR} \qquad (12)$$

ANALYSIS: ELLIPSE PLUS EROSION OR CORROSION

Now consider a stress concentration, k, associated with a very small additional stress concentrator at the end of the major axis of the ellipse. Following the previous analytical sequence it is possible to produce a more general form of eqns (10) and (12).

The hoop stress arising from the initial ellipse given in eqn (4) is now concentrated by a further factor of k giving:

$$\sigma_\theta = pk(1+2c)\frac{(b^2+a^2)}{(b^2-a^2)} \qquad (13)$$

However, somewhat paradoxically, the stress at the end of the major axis of any hole symmetric about its major axis subjected to remote uniaxial stress σ_t parallel to the major axis is still given by [6], [7]:

$$\sigma_{ellipse(t)} = -\sigma_t \qquad (14)$$

This situation is illustrated, together with other relevant solutions, in Figure 3.

Hence, eqn (5) is unchanged by k and

$$\sigma_\theta = \frac{-\beta p a^2}{(b^2 - a^2)} \qquad (15)$$

giving:

$$K_{Lame} = p\frac{[k(1+2c)(b^2 + a^2) - \beta a^2]}{(b^2 - a^2)} \qquad (16)$$

Again, assume that pressure αp acts within the evacuator, where $0 \le \alpha \le 1$

In this case the hoop stress at the end of the major axis of ellipse plus erosion is given by:

$$\sigma_\theta = \alpha p[k(1+2c) - 2] \qquad (17)$$

where the necessary superpositions are again shown in Figure 3..

Note that eqn (17) reduces to eqn (9) for the case where there is no additional stress concentration, i.e. $k = 1$.

Hence the total stress arising from Lamé hoop stress plus Lamé axial stress plus pressure infiltrating the evacuator and erosion may be obtained by summing eqns (13), (15) and (17). If this total stress is normalized using bore hoop stress, eqn (1), the stress concentration factor, $K_{Lame+pressure+erosion}$, is given by:

$$K_{Lame+pressure+erosion} = \frac{b^2[k(1+2c)(1+\alpha) - 2\alpha] + a^2[k(1+2c)(1-\alpha) - \beta + 2\alpha)]}{(b^2 + a^2)} \qquad (18)$$

If there were pre-existing hoop and axial residual stresses $\sigma_{\theta R}$ and σ_{zR} within the tube, these may be treated in the same fashion as the Lamé stresses. This gives

$$\sigma_{\theta Rconc} = k(1+2c)\sigma_{\theta R} - \sigma_{\theta R} \qquad (19)$$

Note that the value of the additional stress concentration, k is a function of the angle of inclination, γ. For example, when looking along the axis of the crossbore, the crossbore hole will appear circular. If an additional small erosion appears semi-circular, hole and erosion will appear elliptical and semi-elliptical respectively at the bore intersection.

SUMMARY AND CONCLUSIONS

The work reported here employs a sequence of analytic, elastic superpositions to obtain a single equation for determining the maximum stress at the bore of a pressurized thick cylinder intersected by an inclined, circular crossbore hole whose axis intersects that of the cylinder. The crossbore hole may contain a pressure that differs from that in the bore, a situation that pertains in some gun tubes during firing.

The formulation permits the modeling of any plane (constant strain) end condition, including zero strain, open-ends and closed-ends. The crossbore hole may itself have an additional stress concentrator on its boundary; such concentrators may result from erosion, corrosion or manufacturing defects. Finally, preexisting residual stresses, prior to introduction of the crossbore hole, may also be incorporated

Since all solutions assume elastic behavior there is no limit upon the Von Mises stress arising from particular combinations of bore pressure, crossbore pressure, end condition and preexisting residual stress. In such cases it will be necessary to adopt some form of 'capping' of the Von Mises stress, taking into account the specific loading sequence.

The single equation presented applies to the crossbore/bore intersection, but the formulation may be directly extended to model any radial location within the wall.

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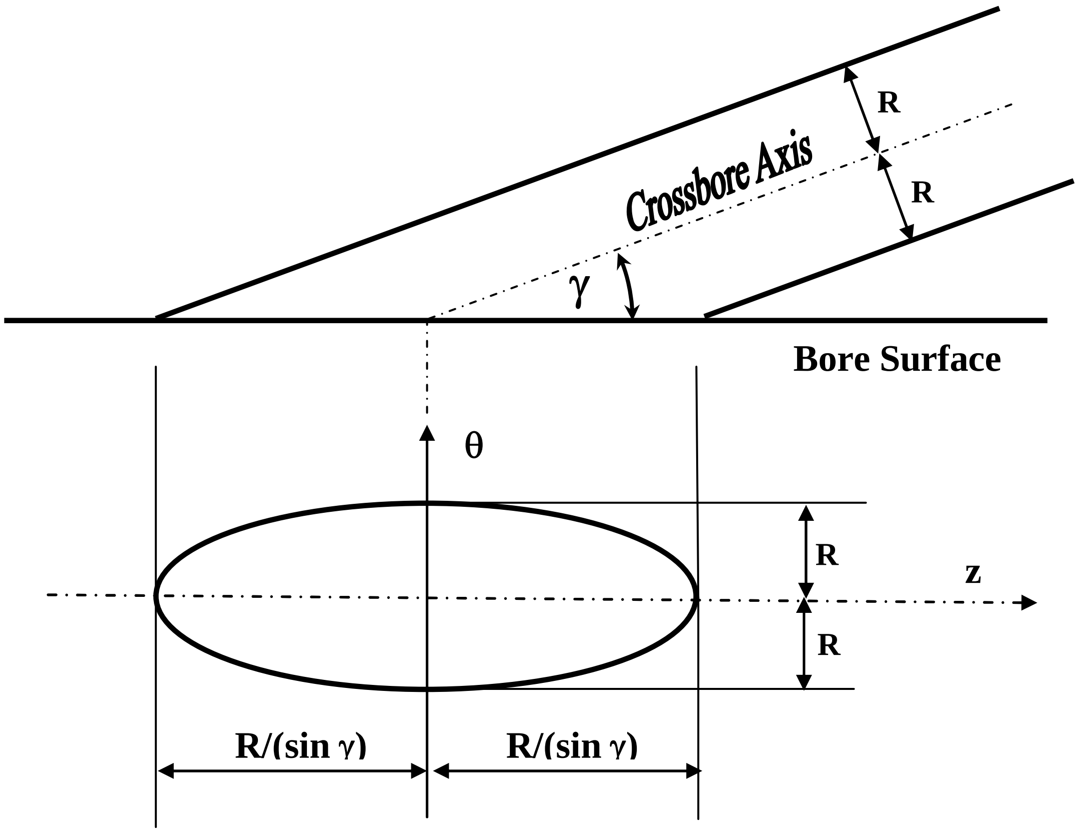


Figure 1: Elliptical Hole Created at Intersection of Inclined Circular Crossbore Hole and Bore of Tube. Hole Radius, R, Angle of Inclination of Hole to Tube Axis

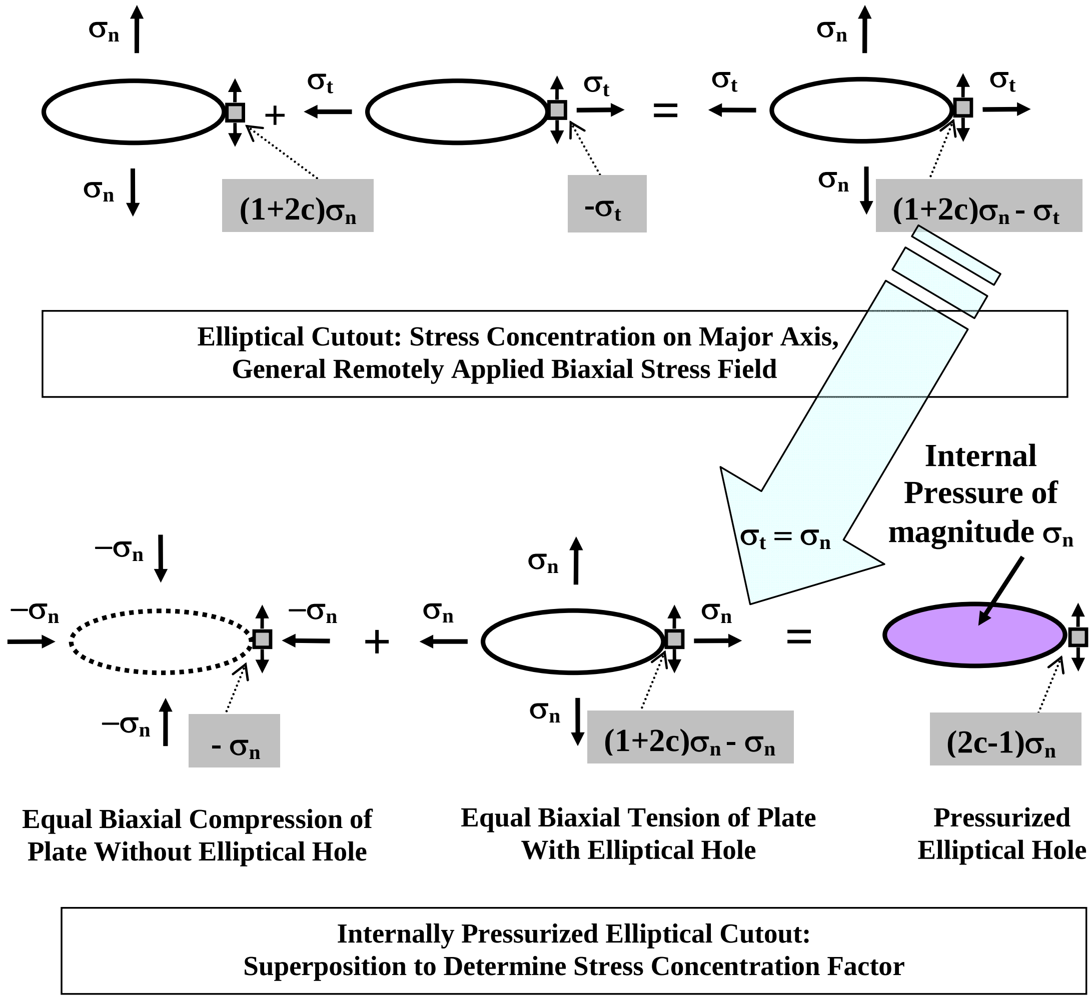
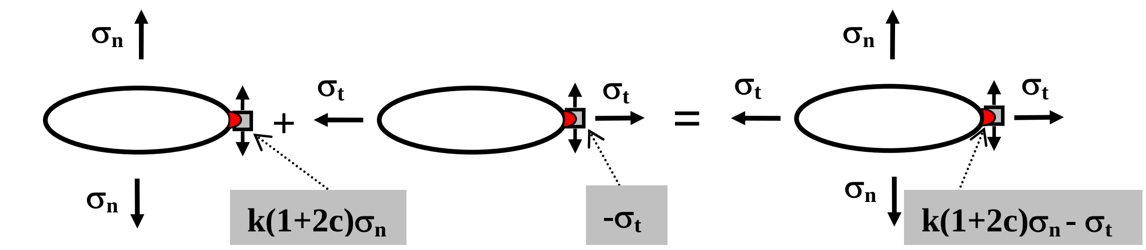
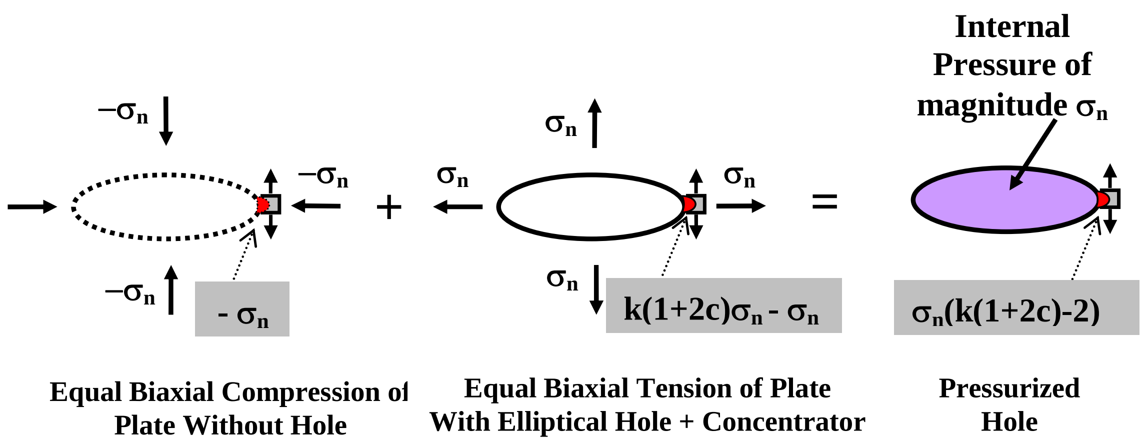


Figure 2: Superposition Sequences for Unpressurized and Pressurized Elliptical Hole at Bore of Thick Cylinder; Eccentricity of Ellipse c



Elliptical Cutout: Stress Concentration on Major Axis + Additional Stress Concentrator (k), General Remotely Applied Biaxial Stress Field



Internally Pressurized Elliptical Cutout + Concentrator (k): Superposition to Determine Stress Concentration Factor

Figure 3: Superposition Sequences for Unpressurized and Pressurized Elliptical Hole plus Additional Stress Concentrator (k) at end of Major Axis; Eccentricity of Ellipse c